PROBLEM OF COSMOLOGICAL SINGULARITY, INFLATIONARY COSMOLOGY AND GAUGE THEORIES OF GRAVITATION

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Abstract. Problem of cosmological singularity is discussed in the framework of gauge theories of gravitation. Generalizing cosmological Friedmann equations (GCFE) for homogeneous isotropic models including scalar fields and usual gravitating matter are introduced. It is shown that by certain restrictions on equation of state of gravitating matter and indefinite parameter of GCFE generic feature of inflationary cosmological models of flat, open and closed type is their regular bouncing character.

PACS numbers: 0420J, 0450, 1115, 9880
KEYWORDS: Cosmological singularity, bounce, inflation, gauge theories of gravity

1 Introduction

Problem of cosmological singularity (PCS) is one of the most principal problems of general relativity theory (GR) and relativistic cosmology. The appearance of singular state with divergent values of energy density and invariants of curvature tensor in cosmological solutions of GR, which limits the existence of the universe in the past, is inevitable according to Hawking-Penrose theorems, if gravitating matter satisfies so-called energy dominance conditions [1, 2]. There were many attempts to resolve PCS in the frame of GR as well as various generalizations of Einstein's theory of gravitation. The radical idea of quantum birth of the Universe was introduced in order to avoid the PCS [3]. A number of regular cosmological solutions was obtained in the frame of metric theories of gravitation and also other theories, in the frame of which gravitation is described by using more general geometry than the Riemannian one (see [4–8] and refs given herein). However, still now we do not have the resolution of PCS. Really, the resolution of PCS means not only containing regular cosmological solutions, but also excluding singular solutions of cosmological equations by using physically reasonable initial conditions. Note that gravitation theory and cosmological equations have to satisfy the correspondence principle with Newton's theory of gravitation and GR in the case of usual gravitating systems with sufficiently small energy densities and weak gravitational fields excluding nonphysical solutions. So metric theories of gravitation based on gravitational Lagrangians including terms quadratic in the curvature tensor lead

to cosmological equations with high derivatives, and although these theories permit to obtain regular cosmological solutions with Friedmann and de Sitter asymptotics, however, they possess nonphysical solutions also. At last time some regular cosmological solutions were found in the frame of superstring theory (brane cosmology), but these solutions also do not resolve the PCS (see for example [7–10]).

Present paper is devoted to study the PCS in the frame of gauge theories of gravitation (GTG), at first of all of the Poincare GTG and metric-affine GTG. Note that GTG are natural generalization of GR by applying the local gauge invariance principle, which is a base of modern theory of fundamental physical interactions (see review [11]). The first attempt to apply the simplest Poincare GTG - the Einstein-Cartan theory — in order to resolve PCS was made in Refs [12, 13], where some nonsingular models with spinning matter were built. Note that these results depend essentially on classical model of spinning matter, and in the case of usual spinless matter the Einstein-Cartan theory is identical to GR and hence singular Friedmann models of GR are its exact solutions. The next step to investigate the PCS in the frame of Poincare GTG was made in Ref.[14] by using sufficiently general gravitational Lagrangian including both a scalar curvature and terms quadratic in the curvature and torsion tensors. Note that in the frame of GTG quadratic in the curvature tensor terms of gravitational Lagrangian do not lead to high derivatives in cosmological equations for homogeneous isotropic models modifying Friedmann cosmological equations of GR (see below). The conclusion about possible existence of limiting energy (mass) density for usual spinless gravitating matter was obtained from deduced cosmological equations leading to regular in metrics cosmological solutions. (Later the possible existence of limiting energy density was discussed in Ref. [15] by modifying cosmological Friedmann equations of GR.) Because cosmological equations of Poincare GTG are valid in the frame of the most general GTG — metric-affine GTG [16, 17], the same conclusions are valid also in metric-affine GTG . Further study of homogeneous isotropic models in GTG has showed that these theories possess important regularizing properties and lead to gravitational repulsion effect nearby limiting energy density, which can be caused by different physical factors [18–21]. Because the behaviour of cosmological models at the beginning of cosmological expansion in GTG depends essentially on equation of state of gravitating matter, we have to know this equation for gravitating matter at extreme conditions in order to build more realistic cosmological models. Unified gauge theories of strong and electroweak interactions with spontaneous symmetry breaking are the modern base of matter description at extreme conditions. Inflationary cosmology as important part of the theory of early Universe was built by using gauge theories of elementary particles [22, 23]. A number of problems of standard Friedmann cosmology were resolved in the frame of inflationary cosmology. Most inflationary cosmological models discussed in literature are singular and their study is given from Planckian time. Essential contribution to energy density in inflationary models is given by so-called gravitating vacuum (connected with scalar fields), for which pressure p and energy density $\rho > 0$ are connected in the following way $p = -\rho$. As it was shown in Refs. [18, 24], in the frame of GTG gravitating vacuum with sufficiently large energy density can lead to the vacuum gravitational repulsion effect (VGRE) in the case of systems including also usual gravitating matter, that allows

to build regular inflationary cosmological models. At first the VGRE was discussed in the frame of Poincare GTG [18] in the case of homogeneous isotropic models including radiation and gravitating vacuum with $\rho = \text{const}$. Regularizing role of gravitating vacuum in GTG was analyzed in Refs [25–28]. Unlike GR where gravitating vacuum can lead to a bounce only in the case of closed cosmological models [29, 30], in GTG the VGRE allows to build bouncing inflationary models of flat, open and closed type. In order to build more realistic inflationary models, homogeneous isotropic models including scalar fields and ultrarelativistic matter were analyzed by applying cosmological equations of GTG [31]. It was shown that by certain conditions GTG permit to build regular inflationary cosmological models of flat, open and closed type with dominating ultrarelativistic matter at a bounce, and the greatest part of cosmological solutions have bouncing character. By taking into account that regular in metrics cosmological solutions in GTG take place in the case of gravitating matter with $p \neq \frac{1}{3}\rho$, we will study below homogeneous isotropic models including besides scalar fields and ultrarelativistic matter also gravitating matter with $p \neq \frac{1}{3}\rho$. In Section 2 cosmological equations of GTG describing such models are introduced. In Section 3 most important general solutions properties of introduced cosmological equations are studied. In Section 4 bouncing character of inflationary cosmological models in GTG is analyzed.

2 Generalized cosmological Friedmann equations in GTG

Homogeneous isotropic models in GTG are described by the following generalized cosmological Friedmann equations (GCFE)

$$\frac{k}{R^2} + \left\{ \frac{d}{dt} \ln \left[R\sqrt{|1-\beta\left(\rho-3p\right)|}\right] \right\}^2 = \frac{8\pi}{3M_p^2}\,\frac{\rho - \frac{\beta}{4}\left(\rho-3p\right)^2}{1-\beta\left(\rho-3p\right)}\,, \tag{1}$$

$$\frac{\left[\dot{R} + R\left(\ln\sqrt{|1-\beta\left(\rho-3p\right)|}\right)^{\cdot}\right]^{\cdot}}{R} = -\frac{4\pi}{3M_p^2}\,\frac{\rho+3p+\frac{\beta}{2}\left(\rho-3p\right)^2}{1-\beta\left(\rho-3p\right)}\,, \tag{2}$$

where $R(t)$ is the scale factor of Robertson-Walker metrics, $k = +1, 0, -1$ for closed, flat, open models respectively, β is indefinite parameter with inverse dimension of energy density, M_p is Planckian mass, a dot denotes differentiation with respect to time[1]. (The system of units with $\hbar = c = 1$ is used). Eqs.(1)–(2) are identical to Friedmann cosmological equations of GR if β=0. At first the GCFE were deduced in Poincare GTG [14], and later it was shown that Eqs.(1)–(2) take place also in metric-affine GTG [16, 17]. From Eqs. (1)–(2) follows the conservation law in usual form

$$\dot{\rho} + 3H\left(\rho + p\right) = 0, \tag{3}$$

[1]Parameter β is defined as $\beta = -\frac{1}{3}\left(16\pi\right)^2 f\, M_p^{-4}$, where f is linear combination of coefficients at terms of gravitational Lagrangian quadratic in the curvature tensor.

where $H = \frac{\dot{R}}{R}$ is the Hubble parameter. Besides cosmological equations (1)–(2) gravitational equations of GTG lead to the following relation for torsion function S and nonmetricity function Q

$$S - \frac{1}{4}Q = -\frac{1}{4}\frac{d}{dt}\ln|1 - \beta(\rho - 3p)|\,. \tag{4}$$

In Poincare GTG $Q = 0$ and Eq. (4) determines the torsion function. In metric-affine GTG there are three kinds of models [17]: in the Riemann-Cartan space-time ($Q = 0$), in the Weyl space-time ($S = 0$), in the Weyl-Cartan space-time ($S \neq 0$, $Q \neq 0$, the function S is proportional to the function Q). The value of $|\beta|^{-1}$ determines the scale of extremely high energy densities. Classical description of gravitational field is valid, if we suppose that $|\beta|^{-1} < 1M_p^4$. The GCFE (1)–(2) coincide practically with Friedmann cosmological equations of GR if the energy density is small $|\beta(\rho - 3p)| \ll 1$ ($p \neq \frac{1}{3}\rho$). The difference between GR and GTG can be essential at extremely high energy densities $|\beta(\rho - 3p)| \gtrsim 1$. Ultrarelativistic matter with equation of state $p = \frac{1}{3}\rho$ is exceptional system because Eqs. (1)–(2) are identical to Friedmann cosmological equations of GR in this case independently on values of energy density. The behaviour of solutions of Eqs. (1)–(2) depends essentially on equation of state of gravitating matter at extreme conditions and on sign of parameter β. By given equation of state $p = p(\rho)$ the GCFE (1)–(2) lead to regular in metrics cosmological solutions in the following cases: 1) $\beta > 0$ and $p < \frac{1}{3}\rho$, 2) $\beta < 0$ and $p > \frac{1}{3}\rho$ [4]. The investigation of models including scalar fields on the base of Eqs (1)–(2) shows that the choice $\beta < 0$ permits to exclude the divergence of time derivative of scalar fields [31]. In connection with this we put below that parameter β is negative and $|\beta|^{-1} < 1 \cdot M_p^4$.

In the case of gravitating vacuum with constant energy density $\rho_v = \text{const} > 0$ the GCFE (1)–(2) are reduced to Friedmann cosmological equations of GR and $S = Q = 0$, this means that de Sitter solutions for metrics with vanishing torsion and nonmetricity are exact solutions of GTG [32] and hence inflationary models can be built in the frame of GTG.

In order to analyze inflationary cosmological models in GTG let us consider systems including scalar field minimally coupled with gravitation and gravitating matter in the form of several components with energy densities ρ_i and pressures $p_i = w_i\rho_i$, where $w_i = \text{const}$ (i is a number of component). Later we will consider two components: ultrarelativistic matter with energy density ρ_r and $w_r = \frac{1}{3}$, and gravitating matter with $w > \frac{1}{3}$. If the interaction between scalar field and all components of gravitating matter is negligible, the energy density ρ and pressure p take the form

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \sum \rho_i, \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi) + \sum w_i\rho_i, \tag{5}$$

where the summation is taken over all components. The conservation law (3) leads to the scalar field equation

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}, \tag{6}$$

where V is scalar field potential and to the conservation laws for all components of gravitating matter

$$\dot{\rho}_i + 3H\rho_i(1 + w_i) = 0. \tag{7}$$

Eqs (7) have integrals in the following form

$$\rho_i R^{3(1+w_i)} = \text{const.} \tag{8}$$

By using Eqs.(5)–(7) the GCFE (1)–(2) can be transformed to the following form

$$\begin{aligned} &\frac{k}{R^2}Z^2 + \left\{H\left[1 - 2\beta(2V+\dot\phi^2) - \frac{1}{2}\beta\sum\rho_i\left(9w_i^2-1\right)\right] - 3\beta V'\dot\phi\right\}^2 \\ &\qquad = \frac{8\pi}{3M_p^2}\left\{\sum\rho_i + \frac{1}{2}\dot\phi^2 + V - \frac{1}{4}\beta\left[4V - \dot\phi^2 - \sum\rho_i\left(3w_i-1\right)\right]^2\right\}Z, \end{aligned} \tag{9}$$

$$\begin{aligned} &\dot H\left\{1 - 2\beta\left[2V + \dot\phi^2 + \frac{1}{4}\sum\rho_i\left(9w_i^2-1\right)\right]\right\}Z \\ &\quad + H^2\left\{\left[1 - 4\beta(V - 4\dot\phi^2) - 4\beta\sum\rho_i\left(1 - \frac{9}{8}w_i - \frac{9}{2}w_i^2 - \frac{27}{8}w_i^3\right)\right]Z \right. \\ &\quad \left. -\frac{9}{2}\beta^2\left[\sum\rho_i\left(1 - 2w_i - 3w_i^2\right) - 2\dot\phi^2\right]^2\right\} \\ &\quad + 12\beta H\dot\phi V'\left\{1 - 2\beta\left[2V + \dot\phi^2 + \frac{1}{4}\sum\rho_i(9w_i^2-1)\right]\right\} \\ &\quad - 3\beta\left[(V''\dot\phi^2 - V'^2)Z + 6\beta\dot\phi^2V'^2\right] \\ &\quad = \frac{8\pi}{3M_p^2}\left\{V - \dot\phi^2 - \frac{1}{2}\sum\rho_i(1+3w_i) - \frac{1}{4}\beta\left[4V - \dot\phi^2 - \sum\rho_i(3w_i-1)\right]^2\right\}Z, \end{aligned} \tag{10}$$

where $Z = 1 - \beta\left[4V - \dot\phi^2 - \sum\rho_i(3w_i-1)\right]$, $V' = \dfrac{dV}{d\phi}$, $V'' = \dfrac{d^2V}{d\phi^2}$. Relation (4) takes the form

$$S - \frac{1}{4}Q = \frac{3\beta}{2}\,\frac{H\left[\dot\phi^2 + \frac{1}{2}\sum\rho_i\left(3w_i^2 + 2w_i - 1\right)\right] + V'\dot\phi}{Z}. \tag{11}$$

Unlike GR the cosmological equation (9) leads to essential restrictions on admissible values of scalar field and gravitating matter. Imposing $\beta < 0$, we obtain from Eq. (9) in the case $k = 0, +1$

$$Z \geq 0 \qquad \text{or} \qquad \dot\phi^2 \leq 4V + |\beta|^{-1} - \sum\rho_i\left(3w_i - 1\right). \tag{12}$$

Inequality (12) is valid also for open models discussed below. The region Σ of admissible values of scalar field ϕ, time derivative $\dot\phi$ and energy densities ρ_i (excluding ρ_r) in space P of these variables determined by (12) is limited by bounds $L_\pm$

$$\dot\phi = \pm\left[4V + |\beta|^{-1} - \sum\rho_i(3w_i-1)\right]^{\frac{1}{2}}. \tag{13}$$

From Eq. (9) the Hubble parameter on the bounds $L_\pm$ is equal to

$$H = \frac{3\beta V'\dot\phi}{1 - 2\beta(2V + \dot\phi^2) - \frac{1}{2}\beta\sum\rho_i(9w_i^2-1)}. \tag{14}$$

According to (14) the right-hand part of Eq. (11) is equal to $\frac{1}{2}H$, this means that the torsion (nonmetricity) will be regular, if the Hubble parameter is regular. In Sections 3-4 our main attention will be turned to study properties of solutions of GCFE (9)–(10).

3 Some general properties of GCFE solutions for inflationary models

Let us consider the most important general properties of cosmological solutions of GCFE (9)–(10). At first, note by given initial conditions for scalar field (ϕ, $\dot{\phi}$) and values of R and ρ_i there are two different solutions corresponding to two values of the Hubble parameter following from Eq. (9):

$$H_{\pm} = \frac{3\beta V'\dot{\phi} \pm \sqrt{D}}{1 - 2\beta(2V + \dot{\phi}^2) - \frac{1}{2}\beta \sum \rho_i (9w_i^2 - 1)}, \tag{15}$$

where

$$D = \frac{8\pi}{3M_p^2}\left\{\sum \rho_i + \frac{1}{2}\dot{\phi}^2 + V - \frac{1}{4}\beta\left[4V - \dot{\phi}^2 - \sum \rho_i\left(3w_i - 1\right)\right]^2\right\} Z - \frac{k}{R^2} Z^2 \geq 0 \tag{16}$$

Unlike GR, the values of H_+ and H_- in GTG are sign-variable and, hence, both solutions corresponding to H_+ and H_- can describe the expansion as well as the compression in dependence on their sign. Below we will call solutions of GCFE corresponding to H_+ and H_- as H_+-solutions and H_--solutions respectively. In points of bounds $L_\pm$ we have $D = 0$, $H_+ = H_-$ and the Hubble parameter is determined by (14). Eqs. (9)–(10) are satisfied on the bounds $L_\pm$, corresponding solutions of GCFE – $L_\pm$–solutions – are their particular solutions; H_-–solutions reach the bounds $L_\pm$ and H_+-solutions originate from them (see below). By using Eqs. (15)–(16) it is easy to shown, that in points of bounds $L_\pm$ the derivatives $\dot{H}_+$ and $\dot{H}_-$ are equal: $\lim \dot{H}_+ = \lim \dot{H}_-$ at $Z \to 0$. As result, we have the smooth transition from H_--solution to H_+-solution on bounds $L_\pm$. At the same time, the value of the time derivative of $\dot{H}$ for $L_\pm$-solutions according to (14) is not equal to $\lim \dot{H}_\pm$ at $Z \to 0$, and by transition from H_--solution to $L_\pm$-solutions and from $L_\pm$-solution to H_+-solution a finite jump of the derivative $\dot{H}$ takes place. Note, that according to Eq. (11) the functions S and Q have the following asymptotics at $Z \to 0$: $\left|S - \frac{1}{4}Q\right| \sim Z^{-1/2}$ for H^+- and H_--solutions.

In order to study the behaviour of cosmological models at the beginning of cosmological expansion, let us analyze extreme points for the scale factor $R(t)$: $R_0 = R(0)$, $H_0 = H(0) = 0$. Denoting values of quantities at $t = 0$ by means of index "0", we obtain

from (9)–(10):

$$\frac{k}{R_0^2}Z_0^2+9\beta^2{V'}_0^2\dot{\phi}_0^2$$
$$=\frac{8\pi}{3M_p^2}\left\{\sum\rho_{i0}+\frac{1}{2}\dot{\phi_0}^2+V_0-\frac{1}{4}\beta\left[4V_0-\dot{\phi}_0^2-\sum\rho_{i0}\left(3w_i-1\right)\right]^2\right\}Z_0, \qquad (17)$$

$$\dot{H}_0=\left\{\frac{8\pi}{3M_p^2}\left[V_0-\dot{\phi}_0^2-\frac{1}{2}\sum\rho_{i0}\left(1+3w_i\right)-\frac{1}{4}\beta\left(4V_0-\dot{\phi}_0^2-\sum\rho_{i0}\left(3w_i-1\right)\right)^2\right]Z_0\right.$$
$$\left.+3\beta\left[({V''}_0\dot{\phi}_0^2-{V'}_0^2)Z_0+6\beta\dot{\phi}_0^2{V'}_0^2\right]\right\}$$
$$\times\left\{1-2\beta\left[2V_0+\dot{\phi}_0^2+\frac{1}{4}\sum\rho_{i0}\left(9w_i^2-1\right)\right]\right\}^{-1}Z_0^{-1}, \qquad (18)$$

where $Z_0=1-\beta\left[4V_0-\dot{\phi}_0^2-\sum\rho_{i0}\left(3w_i-1\right)\right]$. A bounce point is described by Eq. (17), if the value of $\dot{H}_0$ is positive. By using Eq.(17) we can rewrite the expression of $\dot{H}_0$ in the form

$$\dot{H}_0=\left\{\frac{8\pi}{M_p^2}\left[V_0+\frac{1}{2}\sum\rho_{i0}\left(1-w_i\right)-\frac{1}{4}\beta\left(4V_0-\dot{\phi}_0^2-\sum\rho_{i0}\left(3w_i-1\right)\right)^2\right]\right.$$
$$\left.+3\beta({V''}_0\dot{\phi}_0^2-{V'}_0^2)-\frac{2k}{R_0^2}Z_0\right\}\left\{1-2\beta\left[2V_0+\dot{\phi}_0^2+\frac{1}{4}\sum\rho_{i0}\left(9w_i^2-1\right)\right]\right\}^{-1}. \qquad (19)$$

We see from (19) unlike GR the presence of gravitating matter (with $w_i \leq 1$) does not prevent from the bounce realization[2]. In the case of various scalar field potentials applying in inflationary cosmology Eq.(17) determines in space P so-called "bounce surfaces" depending on parameter β and energy density of ultrarelativistic matter parametrically. In the case of closed and open models families of bounce surfaces depend also on the scale factor R_0.By giving concrete form of potential V and choosing values of R_0, ϕ_0, $\dot{\phi}_0$ and ρ_{i0} at a bounce, we can obtain numerically particular bouncing solutions of GCFE for various values of parameter β.

The analysis of GCFE shows, that properties of cosmological solutions depend essentially on parameter β, i.e. on the scale of extremely high energy densities. From physical point of view interesting results can be obtained, if the value of $|\beta|^{-1}$ is much less than the Planckian energy density [33], i.e. in the case of large in module values of parameter β (by imposing $M_p=1$). In order to investigate cosmological solutions at the beginning of cosmological expansion in this case, let us consider the GCFE by supposing that

$$\left|\beta\left[4V-\dot{\phi}^2-\sum\rho_i(3w_i-1)\right]\right|\gg 1,$$
$$\sum\rho_i+\frac{1}{2}\dot{\phi}^2+V\ll|\beta|\left[4V-\dot{\phi}^2-\sum\rho_i(3w_i-1)\right]^2. \qquad (20)$$

[2]In GR a bounce is possible only in closed models if the following condition $V_0-\dot{\phi}_0^2-\frac{1}{2}\sum\rho_{i0}(1+3w_i)>0$ takes place.

Note that the second condition (20) does not exclude that ultrarelativistic matter energy density can dominate at a bounce. We obtain:

$$\frac{k}{R^2}+\frac{\left\{2H\left[2V+\dot{\phi}^2+\frac{1}{4}\sum\rho_i(9w_i^2-1)\right]+3V'\dot{\phi}\right\}^2}{\left[4V-\dot{\phi}^2-\sum\rho_i(3w_i-1)\right]^2}$$
$$=\frac{2\pi}{3M_p^2}\left[4V-\dot{\phi}^2-\sum\rho_i(3w_i-1)\right], \quad (21)$$

$$\dot{H}\left[2V+\dot{\phi}^2+\frac{1}{4}\sum\rho_i\left(9w_i^2-1\right)\right]\left[4V-\dot{\phi}^2-\sum\rho_i\left(3w_i-1\right)\right]+$$
$$H^2\left\{2\left[V-4\dot{\phi}_2+\sum\rho_i\left(1-\frac{9}{8}w_i-\frac{9}{2}w_i^2-\frac{27}{8}w_i^3\right)\right]\left[4V-\dot{\phi}^2-\sum\rho_i\left(3w_i-1\right)\right]\right.$$
$$\left.-\frac{9}{4}\left[2\dot{\phi}^2-\sum\rho_i\left(1-2w_i-3w_i^2\right)\right]^2\right\}-12HV'\dot{\phi}\left[2V+\dot{\phi}^2+\frac{1}{4}\sum\rho_i\left(9w_i^2-1\right)\right]$$
$$+\frac{3}{2}\left(V''\dot{\phi}^2-V'^2\right)\left[4V-\dot{\phi}^2-\sum\rho_i\left(3w_i-1\right)\right]-9V'^2\dot{\phi}^2$$
$$=\frac{\pi}{3M_p^2}\left[4V-\dot{\phi}^2-\sum\rho_i\left(3w_i-1\right)\right]^3. \quad (22)$$

Eqs. (21)–(22) do not include radiation energy density, which does not have influence on the dynamics of inflationary models in the case under consideration (although, as it was noted above, the contribution of ultrarelativistic matter to energy density can be essentially greater in comparison with scalar field and other components of gravitating matter), moreover Eqs. (21)–(22) do not contain the parameter β. According to Eq. (21) the Hubble parameter in considered approximation is equal to

$$H_\pm=\frac{-3V'\dot{\phi}\pm\left|4V-\dot{\phi}^2-\sum\rho_i\left(3w_i-1\right)\right|\sqrt{\frac{2\pi}{3M_p^2}\left[4V-\dot{\phi}^2-\sum\rho_i(3w_i-1)\right]-\frac{k}{R^2}}}{2\left[2V+\dot{\phi}^2+\frac{1}{4}\sum\rho_i(9w_i^2-1)\right]}, \quad (23)$$

and extreme points of the scale factor are determined by the following condition

$$\frac{k}{R_0^2}+9\left[\frac{V_0'\dot{\phi}_0}{4V_0-\dot{\phi}_0^2-\sum\rho_{i0}\left(3w_i-1\right)}\right]^2=\frac{2\pi}{3M_p^2}\left[4V_0-\dot{\phi}_0^2-\sum\rho_{i0}\left(3w_i-1\right)\right]. \quad (24)$$

From Eq. (22) the time derivative of the Hubble parameter at extreme points is

$$\dot{H}_0 = \left\{ \frac{\pi}{3M_p^2} \left[4V_0 - \dot{\phi}_0^2 - \sum \rho_{i0}\left(3w_i - 1\right)\right]^2 + \frac{3}{2}\left(V_0'^2 - V_0''\dot{\phi}_0^2\right) \right.$$
$$\left. +9V_0'^2\dot{\phi}_0^2 \left[4V_0 - \dot{\phi}_0^2 - \sum \rho_{i0}\left(3w_i - 1\right)\right]^{-1}\right\} \left[2V_0 + \dot{\phi}_0^2 + \frac{1}{4}\sum \rho_{i0}\left(9w_i^2 - 1\right)\right]^{-1}. \quad (25)$$

or according to Eq. (24) we can rewrite the expression $\dot{H}_0$ in the following form

$$\dot{H}_0 = \frac{1}{2}\left\{ \frac{27V'^2_0\dot{\phi}_0^2}{4V_0 - \dot{\phi}_0^2 - \sum \rho_{i0}\left(3w_i - 1\right)} + 3\left(V_0'^2 - V_0''\dot{\phi}_0^2\right) \right.$$
$$\left. +\frac{k}{R_0^2}\left[4V_0 - \dot{\phi}_0^2 - \sum \rho_{i0}\left(3w_i - 1\right)\right]\right\} \left[2V_0 + \dot{\phi}_0^2 + \frac{1}{4}\sum \rho_{i0}\left(9w_i^2 - 1\right)\right]^{-1}. \quad (26)$$

Obviously Eqs. (24)–(25) correspond to (17)–(18) in considered approximation.

Now in order to investigate inflationary cosmological models in GTG we will analyze models including scalar fields with positive potentials[3] $V > 0$, ultrarelativistic matter and component of gravitating matter with energy density ρ_1 and $w_1 > \frac{1}{3}$.

4 Analysis of inflationary cosmological models in GTG

At first let us consider models including scalar field and ultrarelativistic matter studied in Ref. [31]. In this case the space P is reduced to the plane of variables $(\phi, \dot{\phi})$, bounds $L_\pm$ — to two curves $\dot{\phi} = \pm\left(4V + |\beta|^{-1}\right)^{\frac{1}{2}}$ and bounce surfaces — to corresponding bounce curves on this plane. As it was shown in Ref.[31] the greatest part of inflationary cosmological solutions in the case under consideration have regular bouncing character, although singular solutions exist also because of divergence of particular $L_\pm$–solutions. To analyze regular bouncing solutions we have to examine bounce curves and $H_\pm$–functions defined by Eq.(15) on the plane P. Bounce curves have simple form, if the scale of extremely high energy densities is much smaller than the Planckian energy density. Then according to (24) bounce curves do not depend on parameter β and bounce curves of flat models are two curves B_1 and B_2 determined by equation

$$4V_0 - \dot{\phi}_0^2 = 3\left(\frac{M_p^2}{2\pi}\,V_0'^2\,\dot{\phi}_0^2\right)^{\frac{1}{3}}$$

on the plane P, which are situated near bounds L_+ and L_- respectively.[4] Each of two curves $B_{1,2}$ contains two parts corresponding to vanishing of H_+ or H_- and denoting by

[3] We do not consider in present paper negative scalar field potentials [35].

[4] The neighbourhood of origin of coordinates on the plane P is not considered in this approximation, the behavior of bounce curves near origin of coordinates was examined in Ref. [34], where scalar fields superdense gravitating systems were discussed.

(B_{1+}, B_{2+}) and (B_{1-}, B_{2-}) respectively. If V' is positive (negative) in quadrants 1 and 4 (2 and 3) on the plane P, the bounce will take place in points of bounce curves B_{1+} and B_{2+} (B_{1-} and B_{2-}) in quadrants 1 and 3 (2 and 4) for H_+-solutions (H_--solutions) (see Fig. 1). To analyze flat bouncing models we have to take into account that besides regions

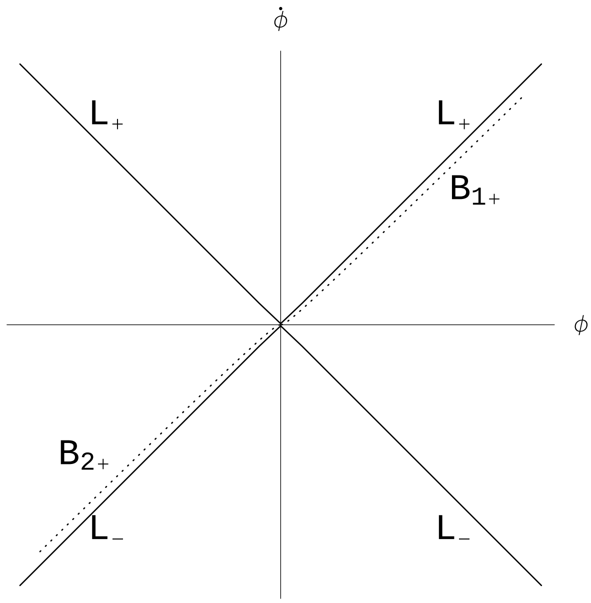

Figure 1: Bounce curves for H_+-solutions and H_--solutions in the case of potential $V = \frac{1}{2}m^2\phi^2$.

lying between bounds $L_\pm$ and corresponding bounce curves on the plane P the sign of values H_+ and H_- for applying potentials is normal: $H_+ > 0$, $H_- < 0$. The Hubble parameter H_+ is negative in regions between curves (L_+ and B_{1+}), (L_- and B_{2+}), and the value of H_- is positive in regions between curves (L_+ and B_{1-}), (L_- and B_{2-}). Any regular cosmological solution has to include both H_-- and H_+-solution. The regular transition from H_--solution to H_+-solution takes place on the bounds $L_\pm$ where $H_+ = H_-$. If H_+- and H_--solution have with $L_\pm$-bounds one common point, corresponding bouncing solution is regular in metrics, the Hubble parameter H and its time derivative. If H_+- and H_--solution are glued with particular L_+- or L_--solution in different points of bounce, corresponding bouncing solution is regular in $R(t)$ and $H(t)$, but the derivative $\dot{H}$ in points of gluing has a finite jump. In the case of open and closed models Eq. (24) ($\rho_i = 0$, with the exception $\rho_r \neq 0$) determines 1-parametric family of bounce curves with parameter R_0. Bounce curves of closed models are situated on the plane P in region between two bounce curves B_1 and B_2 of flat models, and in the case of open models bounce curves are situated in two regions between the curves: L_+ and B_1, L_- and B_2. Because the behaviour of bounce curves for open models is like to that for flat models, the situation concerning bouncing inflationary solutions in the case of open models is the same as described above situation for flat models. Unlike flat and open models, for which $H_+ = H_-$ only in points of bounds $L_\pm$ and regular inflationary models can

be built if H_+– and H_-–solutions reach bounds $L_\pm$, in the case of closed models the regular transition from H_--solution to H_+-solution is possible without reaching the bounds $L_\pm$. It is because by certain value of R according to (16) we have $H_+ = H_-$ in the case, if $Z \neq 0$. Such models are regular also in torsion and/or nonmetricity. Regular inflationary solution of such type was considered in Ref.[33].

However, in discussed case there are also singular solutions. At first note that particular $L_\pm$–solutions are singular. Because bounds $L_\pm$ for applying scalar field potentials tend to infinity, we have that scalar field satisfying on the bounds the equation $\ddot{\phi} = 2V'$ diverges in the past (quadrants 2 and 4) and in the future (quadrants 1 and 3). In consequence of this any solution including H_+-solution (or H_-–solution) glued with one of the bounds $L_\pm$ is singular. Hence the problem of excluding of singular solutions is connected with regularization of particular $L_\pm$–solutions.

In general case, when approximation (20) is not valid, bounce curves of cosmological models including scalar fields and ultrarelativistic matter determined by Eq.(17) depend on parameter β . By certain value of β we have 1-parametric family of bounce curves with parameter ρ_{r0} for flat models, and we have 2-parametric families of bounce curves for closed and open models with parameters R_0 and ρ_{r0}. The situation concerning cosmological solutions of Eqs. (9)–(10) does not change.

Now let us analyze models including gravitating matter with energy density ρ_1 and $w_1 > \frac{1}{3}$ besides scalar field and ultrarelativistic matter. The bounds $L_\pm$ in this case depend on energy density ρ_1 and they change by evolution of models

$$\dot{\phi} = \pm\sqrt{4V + |\beta|^{-1} - \rho_1 \left(3w_1 - 1\right)}\,. \tag{27}$$

The Hubble parameter on bounds $L_\pm$ according to (14) is equal to

$$H = \frac{3\beta V'\dot{\phi}}{1 - 2\beta(2V + \dot{\phi}^2) - \frac{1}{2}\beta\rho_1(9w_1^2 - 1)}. \tag{28}$$

Then the equation of scalar field (6) on bounds $L_\pm$ because of (27)–(28) takes the following form

$$\ddot{\phi} + \frac{V'\left(1 - 4\beta V + \dfrac{5 + w_1}{1 + w_1}\,\beta\dot{\phi}^2\right)}{1 - 4\beta V - \dfrac{1 - w_1}{1 + w_1}\beta\dot{\phi}^2} = 0. \tag{29}$$

If $\frac{1}{3} < w_1 \leq 1$ and $\beta < 0$, the denominator of a fraction in (29) is positive, and for various potentials V applying in inflationary cosmology (in particular, $V = \frac{1}{2}m^2\phi^2$, $V = \frac{1}{4}\lambda\phi^4$ etc.) Eq. (29) describes finite variations of scalar field between positive maximum and negative minimum values of ϕ at $\dot{\phi} = 0$ (see Fig. 2). This means that particular solution of GCFE becomes regular, and it has bouncing character; a bounce takes place in points where $\dot{\phi} = 0$ (according to (14) and (19) we have in these points $H = 0$ and $\dot{H} > 0$). As result, all inflationary cosmological solutions of GCFE are regular. The analysis of inflationary

solutions reaching the bounds $L_\pm$ by numerical integration of Eqs. (11) and (6) is difficult, because the coefficient at $\dot{H}$ in Eq. (11) tends to zero at $Z \to 0$. Note that bouncing character have solutions not only in classical region, where scalar field potential, kinetic energy density of scalar field and energy density of gravitating matter do not exceed the Planckian energy density, but also in regions, where classical restrictions are not fulfilled and according to accepted opinion quantum gravitational effects can be essential.

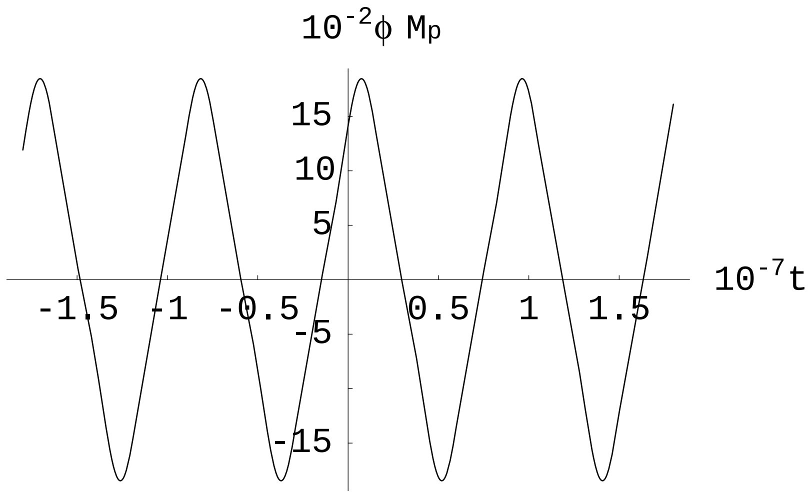

Figure 2: Numerical solution of Eq. (29) for $V = \frac{1}{2}m^2\phi^2$ ($m = 10^{-6}M_p$, $w = 1$, $\beta = -10^6 M_p^{-4}$).

As illustration of obtained results we will consider particular bouncing cosmological inflationary solution for flat model by using scalar field potential in the form $V = \frac{1}{2}m^2\phi^2$ ($m = 10^{-6}M_p$). The solution was obtained by numerical integration of Eqs. (6), (10) and by choosing in accordance with Eq.(17) (or (24)) the following initial conditions at a bounce: $\phi_0 = \sqrt{2}\,10^3\,M_p$, $\dot{\phi}_0 = 10^{-3}M_p^2$, $\rho_{10} = 1.4999 \cdot 10^{-6}M_p^4$ ($w = 1$, $\beta = -10^{14}M_p^{-4}$); in accordance with (8) the formula $\rho_1(t) = \rho_{10}\dfrac{R_0^6}{R^6(t)}$ was used, initial value of R_0 can be arbitrary. A bouncing solution includes: quasi-de-Sitter stage of compression, the stage of transition from compression to expansion, quasi-de-Sitter inflationary stage, stage after inflation. The dynamics of the Hubble parameter and scalar field is presented for different stages of obtained bouncing solution in Figures 3–5 (by choosing $M_p = 1$). The transition stage from compression to expansion (Fig. 3) is essentially asymmetric with respect to the point $t = 0$ because of $\dot{\phi}_0 \neq 0$. In course of transition stage the Hubble parameter changes from maximum in module negative value at the end of compression stage to maximum positive value at the beginning of expansion stage. The scalar field changes linearly at transition stage, the derivative $\dot{\phi}$ grows at first from positive value $\dot{\phi}_1 \sim 1.6 \cdot 10^{-7}$ to maximum value $\dot{\phi} \sim \dot{\phi}_0$ and then decreases to negative value $\dot{\phi}_2 \sim -1.6 \cdot 10^{-7}$. Quasi-de-Sitter inflationary stage and quasi-de-Sitter compression stage are presented in Fig. 4. Although the GCFE (9)–(10) and

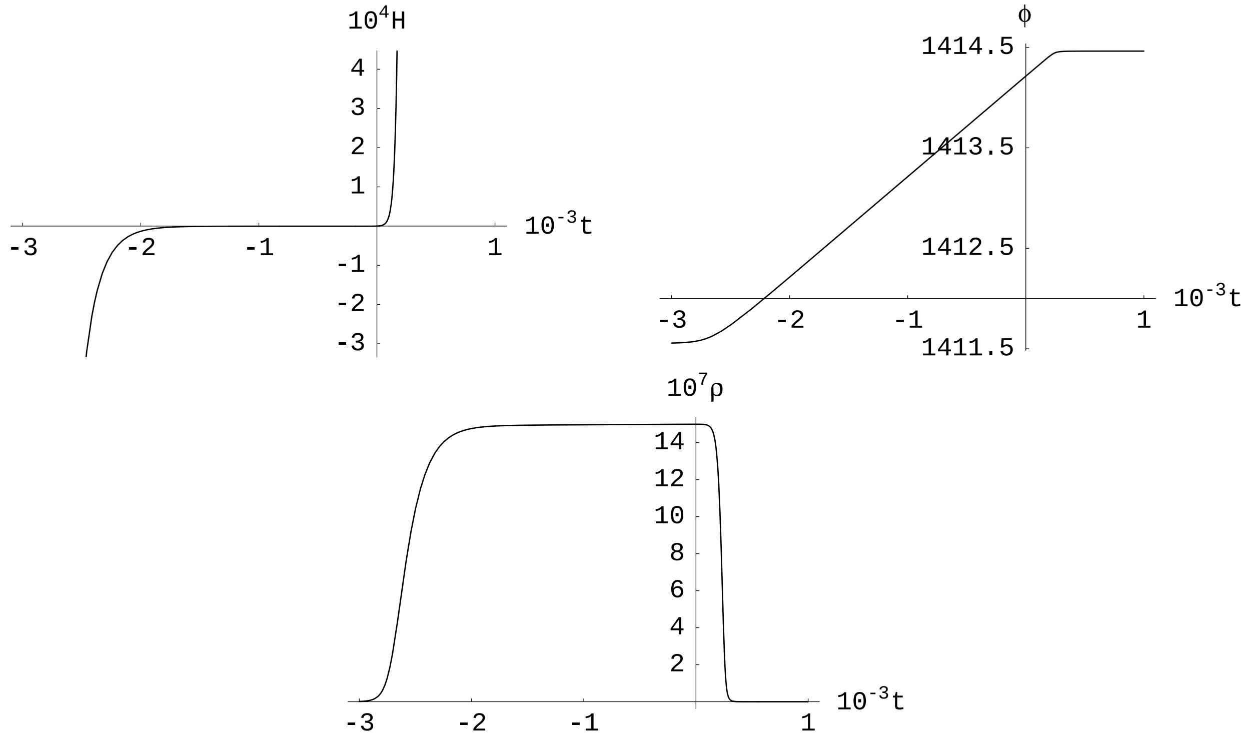

Figure 3: The stage of transition from compression to expansion.

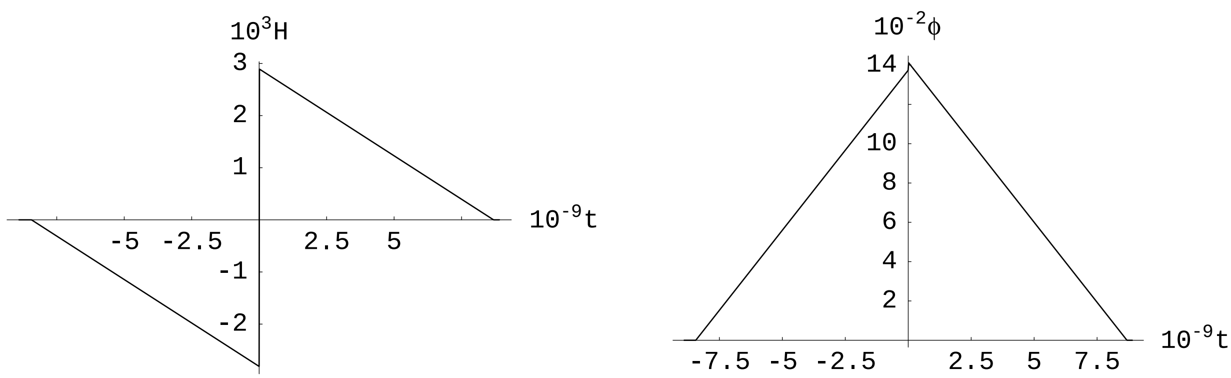

Figure 4: Quasi-de-Sitter stage of compression and inflationary stage.

their approximation (21)–(22) have different structure from cosmological Friedmann equations of GR, like GR the time dependence of functions $H(t)$ and $\phi(t)$ at compression and inflationary stages is linear. The amplitude and frequency of oscillating scalar field after inflation (Fig. 5) are different than that of GR, this means that approximation of small energy densities $\left|\beta\left(4V-\dot{\phi}^2-2\rho_1\right)\right| \ll 1$ at the beginning of this stage is not valid; however, the approximation (21)–(22) is not valid also because of dependence on parameter β of oscilla-

tions characteristics, namely, amplitude and frequency of scalar field oscillations decrease by increasing of $|\beta|$ [33]. The behaviour of the Hubble parameter after inflation is also noneinsteinian, at first the Hubble parameter oscillates near the value $H = 0$, and later the Hubble parameter becomes positive and decreases with the time like in GR. Before quasi-de-Sitter compression stage there are also oscillations of the Hubble parameter and scalar field not presented in Figures 3–5. Ultrarelativistic matter, which could dominate at a bounce as well as another component of gravitating matter have negligibly small energy densities at quasi-de Sitter stages. At the same time the gravitating matter could be at compression stage in more realistic bouncing models, and scalar fields could appear only at certain stage of cosmological compression. As it follows from our consideration regular character of such inflationary cosmological models has to be ensured by cosmological equations of GTG.

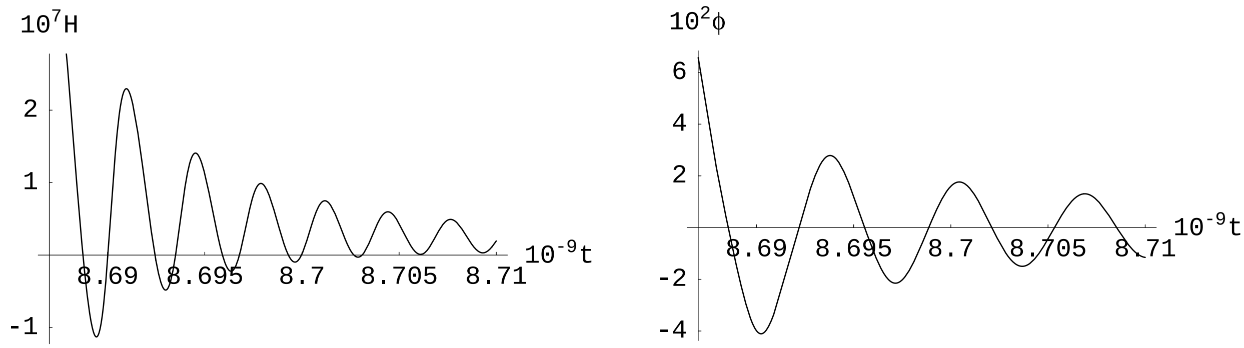

Figure 5: The stage after inflation.

5 Conclusion

As it is shown in our paper, GTG permit to build regular inflationary cosmology, if gravitating matter at extreme conditions satisfies the following restriction $\frac{1}{3}\rho < p \le \rho$ and indefinite parameter β is negative. All inflationary cosmological solutions for flat, open and closed models are regular in metrics and the Hubble parameter. The presence of scalar fields leading to appearance of inflation in cosmological models changes essentially the structure of GCFE, as result a family of closed models regular in metrics, the Hubble parameter and torsion and/or nonmetricity appears. To build realistic inflationary cosmological models not limited in the time we have to know the change of equation of state of gravitating matter by evolution of the Universe.

Acknowledgements

I am grateful to Alexander Garkun and Andrey Minkevich for help by preparation of this paper.

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